# Optimal Placement of Phasor Measurement Units for State Estimation using Artificial Intelligence Techniques 

Ganga Reddy Tankasala, Sridhar Sanisetty, Varun Kumar Vala<br>Abstract- This paper deals with a study carried out to determine the optimal locations of phasor measurement units (PMUs) for a given pow er system. Pow er systems are rapidly becoming populated by PMUs as they provide valuable phasor information of voltages and currents for protection, operation and control of power systems during normal and abnormal operation. This paper focuses on the use of PMU measurements in state estimators. The principle objective was to investigate methods of determining optimal locations for PMUs so that the entire power system is observable. The recently developed AI techniques, like Genetic Algorithm and artificial bee colony techniques are applied to find out the optimal placement of PMUs for various systems. It is found that the entire system can be made observable by strategically placing PMUs at one-third of the system buses for a given system.

Index Terms—Artificial Bee colony (ABC), Artificial Intelligence Techniques, Genetic Algorithm (GA), Phasor Measurements Unit (PMU), Pow er system, State estimation

## 1 Introduction

Power systems operation mainly consists of data acquistion, monitoring and controlling of the system. of which the data acquisition and monitoring plays a very important role in its secure operation. Before the advent of the Phasor measurement units (PMU)'s the data i.e, the analog and digital data(statuses) of the circuit breakers from various substations are fed as input to the estimators present in the control center computer. Now with the development of GPS, these devices use a navigational satellite system to synchronize digital sampling at different substations. That summer the data was analysed using modem digital signal processing software.

State estimators of a power grid provide optimal estimates of bus voltage phasors based on the available measurements and knowledge about the network topology. These measurements are commonly provided by the remote terminal units (RTU) at the substations and include real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents. More recently, synchronized phasor measurements have started to become available at selected substations in the system. One of the issues faced by the planning engineers is how to select the best locations to install new PMUs. Earlier work done by Phadke and his co-workers [1-2] introduces the use of PMUs for such applications. This work is later extended to the investigation of optimal location of PMUs where each PMU is assumed to provide voltage and current phasors at its associated bus and all incident branches [3].

It is therefore possible to fully monitor the system by using relatively small number of PMUs much less than the number of buses in the system. This problem isformulated and solved by using a graph theoretic observability analysis and an optimization method based on Artificial bee colony [4]. Possible loss or failure of PMUs is not considered in that study.
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## 2 Optimal Placement Of PMUs

### 2.1 Problem Formulation

A numerical method based on Integer Programming is explained here under to solve our problem. The formulation of problem is shown as below.

For an n-bus system, the PMU placement problem is formulated as follows:

$$
\begin{equation*}
\text { Objective function }(J)=\operatorname{Min}\left(\sum W_{i} * X_{i}\right) \quad \forall \quad i \in \text { Buses } \tag{1}
\end{equation*}
$$

Such that $f(X) \geq 1^{\wedge}$
Where X is a binary decision variable vector, whose entries are defined as:

$$
\begin{align*}
X_{i} & =\{1 \\
& =\left\{\begin{array}{lc}
1 & \text { if a PMU is installed at bus } i \\
& \text { otherwise }
\end{array}\right. \tag{3}
\end{align*}
$$

$W_{i}$ is the cost of the PMU installed at bus $i$.
$f(X)$ is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measure ment set and zero otherwise.
$1^{\wedge}$ is a vector whose entries are all ones.

Inner product of the binary decision variable vector and the cost vector represents the total installation costs of the selected PMUs. Constraint functions ensure full network observability while minimizing the total installation cost of the PMUs.

### 2.2 Constraints

The procedure for building the constraint equations described for three following possible cases is presented

1. No conventional measurement or zero injections
2. Flow measurements and
3. Flow measurements as well as injection measurements (they may be zero injections or measured injections).

Description of the procedure for each case is given using IEEE 14-bus system example for clarification.


Fig 1.IEEE 14-bus system with conventional measurements
The numbers at each node represents the bus number. One can see the grid connections among all the buses.The constraints, categorized in to three classes are discussed below in detail.

## 3 Illustrations

### 3.1 A system with no conventional measurements and/or measurements

In this case, the flow measurement and the zero injection are ignored. In order to form the constraint set, the binary connectivity matrix A, whose entries are defined below, has to be formed first:

Matrix A can be directly obtained from the bus admittance matrix by transforming its entries into binary form. Building the A matrix for the 14-bus system yields:

$$
\begin{align*}
A_{k, m} & =\left\{\begin{array}{lc}
1 & \text { if a } k=m \text { or } k \text { and } m \text { are connected } ; \\
& =\{0 \\
\text { otherwise }
\end{array}\right.
\end{align*}
$$

The A matrix for IEEE 14 bus system shown in fig (1) can be evaluated as

$$
A=\quad\left[\begin{array}{r}
0 \\
0
\end{array} 000000001000011\right.
$$

$$
\left.\begin{array}{lllllllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

The constrants for this case can be formulated as

$$
\begin{equation*}
f(X)=A \cdot X \tag{5}
\end{equation*}
$$

The

$$
\begin{gather*}
f_{i}=A(i,:) \times X(i, 1) \quad \forall i \in \text { Buses of the grid }  \tag{6}\\
f_{1}=\left(X_{1}+X_{2}+X_{5}\right) \geq 1 \\
f_{2}=\left(X_{1}+X_{2}+X_{3}+X_{4}+X_{5}\right) \geq 1 \tag{7}
\end{gather*}
$$

The operator " + " serves as the logical "OR" and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non-zero which means one of atleast 1,2 and 5 numbered buses of eq (6) should be provided with aPMU to make bus1 observable.

Similarly the second constraint implies that one PMU should be installed at any of the buses $1,2,3,4$ or 5 inorder to make bus 2 observable.

### 3.2 A system with atleast one flow measurement or zero injection bus

This case considers the situation where some flow measurements may be present. Existence of flow measurement will lead to the modification of the constraints for buses accordingly. Modification follows the observation that having a flow measurement along a given branch allows the calculation of one of the terminal bus voltage phasors when the other one is known. Hence, the constraint equations associated with the terminal buses of the measured branch can be merged into a single constraint. In the case of the example system, the constraints for buses 5 and 6 are merged into a joint constraint as there is flow measurement in branch between 5 and 6 .

$$
\begin{align*}
& f_{5}=\left(X_{1}+X_{2}+X_{4}+X_{5}+X_{6}\right) \geq 1  \tag{8}\\
& f_{6}=\left(X_{11}+X_{12}+X_{13}+X_{5}+X_{6}\right) \geq 1  \tag{9}\\
& f_{5-6 \text { new }}=f_{5}+f_{6} \\
& \quad=\left(X_{1}+X_{2}+X_{4}+X_{5}+X_{6}+X_{11}+X_{12}+X_{13}\right) \geq 1 \tag{10}
\end{align*}
$$

Which implies that if either one of the voltage phasors at bus 5 or 6 is observable, the other one will be observable.

Remove the $f_{5}$ and $f_{6}$ from function f and them by new constraint $f_{5-6 \text { new }}$.

### 3.3 A system with both injection measurements (some of which may be zero injection pseudo measurements) and flow measurements.

This case considers the most general situation where both injection and flow measurements may be present, but not enough to make the entire system observable. Injection measurements whether they are zero injections or not, are treated the same way. Consider again the same 14 -bus system shown in fig (1), where bus 7 is a zero injection bus. It is easy to see that if the phasor voltages at any three out of the four buses 4 , 7,8 and 9 are known, then the fourth one can be calculated using the Kirchhoff's Current Law applied at bus 7 where the net injected current is known.

One way to treat the injection buses is to modify the constraints associated with theneighboring buses of these buses and form a set of non-linear constraints. This is accomplished as shown below. To treat the zero injection bus 7 in the IEEE 14-bus system, constraints associate with its neighboring buses 4,8 and 9 will be modified as follows.

$$
\begin{align*}
& f_{4}=\left(X_{2}+X_{3}+X_{4}+X_{5}+X_{7}+X_{9}+f_{7} \cdot f_{8} \cdot f_{9}\right) \geq 1  \tag{11}\\
& f_{7}=\left(X_{4}+X_{7}+X_{8}+X_{9}+f_{4} \cdot f_{8} \cdot f_{9}\right) \geq 1  \tag{12}\\
& f_{8}=\left(X_{7}+X_{8}+f_{4} \cdot f_{7} \cdot f_{9}\right) \geq 1  \tag{13}\\
& f_{9}=\left(X_{4}+X_{7}+X_{9}+X_{10}+X_{14}+f_{4} \cdot f_{7} \cdot f_{8}\right) \geq 1 \tag{14}
\end{align*}
$$

The operator '.' in the above equations serves as the logical "AND" Operation. The expressions for $f_{i}$ can be further simplified by using the following properties of the Boolean logical AND (.) and OR (+) operators.

Given two sets $A$ and $B$, where set $A$ is a subset of set $B$, then $A+B=B$ and $A \cdot B=A$

By substituting $f_{7}, f_{8}$ and $f_{9}$ in expression for $f_{4}, f_{4}$ can be written as

$$
f_{4}=\left(X_{2}+X_{3}+X_{4}+X_{5}+X_{7}+X_{9}+X_{8} \cdot X_{10}+X_{8} \cdot X_{14}\right)
$$

Applying similar simplification to other expressions, other constraints can be redefined as

$$
\begin{aligned}
& f_{8}=\left(X_{4}+X_{7}+X_{8}+X_{9}\right) \\
& f_{9}=\left(X_{4}+X_{7}+X_{9}+X_{10}+X_{14}+X_{2} \cdot X_{8}+X_{3} \cdot X_{8}+X_{5} \cdot X_{8}\right)
\end{aligned}
$$

The constraints corresponding to all other buses will remain the same as given in eq (2). The zero injection bus constraint is eliminated as it is taken care of by its neighbors.

## 4 Artificial Bee Colony Algotithm For Optimal Placement Of Phasor Measuremrnt Units

1. Generate n random solutions with in boundaries of the system

> a. $X=$ Boolean (rand (No of solutions, size of solution))
2. Check that random solutions satisfy the inequality
constraints of buses i.e.
a. $f(X)=(A . X) \geq 1^{\wedge}$
b. where A is binary impedance matrix, X is the solution.
3. Calculate the objective function and fitness of each solution
4. Store the best fit as $\mathrm{X}_{\text {best }}$ solution
5. A mutant solution is formed using a randomly selected neighbour
a. If (rand $>0.5$ )
b. $\quad X_{k \text { mutant }}=X_{k}(i) \quad$ OR $\quad X_{j}(i)$
c. Else
d. $\quad X_{k \text { mutant }}=X_{k}(i)$ AND $X j(i)$
e. Where j is the randomly selected neighbour and $i$ is a
f. random parameter. OR and AND refers to Boolean
g. Operators. Rand represents a random number
h. between 0 and 1 .
6. Check for constraints $f(X)=($ A. $X) \geq 1^{\wedge}$. If the constraints are satisfied proceed to step 7 , else move back to step 5 .
7. Replace $X_{k \text { mutant }}$ by $X_{k}$, if the mutant has higher fitness or lower cost of PMU.
8. Repeat the above procedure for all the solutions.
9. Onlooker bee phase (Simple ABC)
10. Probability of each solution is calculated as
a. Probability (i) $=\mathrm{a}$ *fitness ( i )/max (fitness) +b
b. Where $\{a+b=1\}$
11. The solution $X$ is selected if its Probability is greater than a random number.
i. If (rand<probability (i))
ii. Solution is accepted for mutation.
b. Else
c. Solution is discarded for mutation
12. Again the best $X_{\text {best }}$ is determined
13. Replace a $X$ by random $X$ if its trial counter exceeds threshold (Scout bee phase)
14. Repeat the above for max no of iterations
15. The $X_{\text {best }}$ and $F\left(X_{\text {best }}\right)$ are the best solution and Global minimum of the objective function.

## 5 Placement Of PMUs for Various Cases

The integer bases artificial bee colony optimization method is tested on IEEE 14-bus, 30 -bus, 57 -bus and 118-bus systems. Detailed system information and simulation results are presented in the following sub-sections named 5.1, 5.2.5.3 and 5.4.

### 5.1 IEEE 14 bus system



Fig 5.1 IEEE 14 bus system
The above mentioned system is considered for applying the ABC approach to find evaluate the optimal placement of PMU's in order to observe the total system. The results of the simulation are tabulated below.

Table 1. Simulation results of IEEE 14 bus system

| Zero injec- <br> tion buses | Number of <br> branches | Location of <br> PMUs | Total no. of <br> PMUs |
| :--- | :--- | :--- | :--- |
| None | 20 | $2,7,11,13$ | 4 |
| 7 | 20 | $2,6,9$ | 3 |

### 5.2 IEEE 30 bus system



Fig 5.2 IEEE 30 bus system
The IEEE 30 bus system is simulated with and without considering the zero injections and the results are tabulated in the below table.

Table2. Simulation results of IEEE 30 bus system

| Zero <br> injection <br> buses | Number <br> of <br> branches | Location of PMUs | Total <br> no. of <br> PMUs |
| :--- | :--- | :--- | :--- |


| None | 41 | $1,5,9,10,12,15,18,25,27,28$ | 10 |
| :--- | :--- | :--- | :--- |
| $6,9,11$, <br> 25,28 | 41 | $3,5,10,12,18,23,27$ | 7 |

### 5.3 IEEE 57 bus system



Fig 5.3 IEEE 57 bus system
Simulation results for above shown IEEE 57 -bus system with and without considering zero injections are tabulated in the table 5.3.

Table3. Simulation results of IEEE 57 bus system

| Zero injec- <br> tion buses | Number of <br> branches | Location of <br> PMUs | Total no. of <br> PMUs |
| :--- | :--- | :--- | :--- |
| None | 78 | $1,6,13,15,18$, | 19 |
|  |  | $21,22,25,27$, |  |
|  |  | $29,32,34,38$, |  |
| $4,7,11,21,22,46,51$, | 78 | $1,6,9,15,20$, | 13 |
| $24,26,34,36$, |  | $25,27,32,38$, |  |
| $37,39,40,45$, |  | $47,50,53,56$ |  |
| 46,48 |  |  |  |

### 5.4 IEEE 118 bus system

Results of the IEEE 118 bus system simulated with and without considering the zero injections are tabulated below. The data of IEEE 118 bus system is presented in Table no 1 of the Appendice.

Table4. Simulation results of IEEE 14 bus system

| Zero in- <br> jection <br> buses | Number <br> of <br> branches | Location of PMUs | Total no. of <br> PMUs |
| :--- | :--- | :--- | :--- |
| None | 186 | $2,5,9,11,12,17,21$, | 32 |


|  |  | $24,25,28,34,37,40$, |  |
| :--- | :--- | :--- | :--- |
|  |  | $45,49,52,56,62,63$, |  |
|  |  | $68,73,75,77,80,85$, |  |
|  |  | $86,90,94,101,105$, |  |
| $5,9,30$, | 110,114 |  |  |
| $37,38,63$, | $2,8,11,12,15$, | 29 |  |
| $64,68,71$, |  | $19,21,27,31,32,34,40$, |  |
| 81 |  | $45,49,2,56,62,65$, |  |
|  |  | $72,75,77,80,85,86$, |  |
|  |  | $90,94,101,105,110$ |  |

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## 6 Conclusion

The $A B C$ approach for solving a binary mode of optimization gives the desired optimized results succesfully. The ABC approach gurantees a global or near global solution with a properly chosen colony parameters like maximum number of iterations, population size, onlooker bees, employed bees and threshold limit.

## Appendices

The standard IEEE 57 Bus and IEEE 118 Bus systems data are presented in a separate document attached to avoid congestion of data in the paper. The same can be obtained from any IEEE website or other reliable sources.

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